The analysis of signals propagation in transmission lines in
the configuration of CAN-bus controller

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The method of signal propagation analysis in the transmission lines of CAN-buses has been presented in the paper. Its aim is to determine and/or verify of the CAN controllers configuration parameters from the point of view of realistic conditions in the designed network which are often omitted in the standard calculation procedure. The adequate computer programs, to solve the transmission line equations in the area of assumptions, were elaborated. It allows to obtain the practically useful parameters like: characteristic line impedance, signal propagation times, voltage drops and ground potentials, etc, which, in turn, are selection base of right CAN controller sets. This paper contains the review of applied assumptions and examples resulted from the conducted investigations.

1. Introduction

It is generally assumed that the designers of CAN-bus based systems should have sufficient experience and an intuition. Owing to them the designers can properly determine final parameters of the systems like the highest transmission speed, transmission medium type, achievable transmission range etc. at given environment conditions and specified EMC requirements.

Validity of the initial assumptions is usually verified on the base of obtained experimental results; if there is a need the assumptions may be corrected in a recurrent way. Such design approach is time-consuming procedure and, what is even worse, in most situation it does not provide the system optimisation.

It would be therefore strongly desired to develop a method of the selection of CAN-bus operational or functional parameters on the base of mathematical line model. The model should provide a good relation with the real parameters of the lines as well as of all bus nodes.

According to our proposals, the equivalent parameters of line model, well corresponding to the parameters of real layout of bus conductors, are determined by consideration of bus structure and its geometrical parameters with regard to the distribution of bus nodes. From this reason in the proposed bus model some factors were taken into account so far neglected or often simplified during calculations: number of nodes and their location within the line as well as their input and output characteristics. More parameters, like: real propagation times of useful signals and standard disturbances in multiconductor CAN-bus, transmission line types and their spatial configuration (a pair of wires, a pair over the ground plane, twisted pair, shielded pair etc.) were also taken into account.

The final conclusion is, that the proposed method of determination of the configuration of CAN-bus controller, based on the consideration of transmission line parameters, allows to obtain the required properties of the CAN bus at the given conditions.

2. Per-unit-length parameters

The CAN-bus specification doesn’t impose the specific type of conductors for its creation (Fig.1). The fundamental criterion of selection depends then on the parameters, which allow to obtain the expected signals propagation time in the bus, acceptable voltage drops and required EMC properties. It touches both the immunity aspect important in environment with high level of electromagnetic interferences (for example wire shielding and twisting, additional shield of node connections) as well as disturbance emission caused by poor
quality of non-shielded wires (significant asymmetry of particular conductors and insulations) accompanied by placement errors. The type of used wires, their geometry and cross-section area implicate per-unit-length line resistance, insulation leakage, capacitances and self and mutual inductances and wave impedance necessary for detailed analysis. Even the wires from good manufacturers with well known and stable initial parameters in a certain application (e.g. if the wires are placed on the metal supporting the structure of the building) can change these parameters. The problem of parasitic parameters is not observed in the case of homogeneous transmission lines terminated by characteristic impedance except per-unit-length resistance (easy to determine). The real bus having specified resistance and inductance of CAN_H and CAN_L inputs doesn’t meet such assumption. Determination of such parameters in analytical form is very hard and more complicated for increasing number of the line wires. The most often in calculations the close proximity effect should be taken into consideration. It resulting from the placement of many wires in relatively small distances between them. Such located conductors cause the disturbance of homogeneous charge distribution on surfaces of the rest, what decreases the results accuracy. In those situations, the numerical methods are used for the MTL parameters determination. They allow to obtain of the those parameters from electromagnetic field equations with adequate accuracy (despite of simplified assumptions).

[Moment method]

Making assumption that in transmission lines of the CAN-bus the wave of TEM type (or quasi-TEM) is propagated, the moment method (Galerkin method) can be applied for computations. It allows to replace of the integral equations system with linear equations system for particular wires. It can be realised numerically – in easy way – in Math-CAD program with good accuracy. The quantities which characterised electromagnetic field in dielectric medium can be determined from Poisson equation formulated for considered wire system. With assumption of line homogeneity only the mutual configuration of m+1 conductors and dielectrics has an influence on residual parameters. With circular section of conductors it is the most convenient to consider of the cylindrical co-ordinate system with centre in the middle of reference wire.

\[
\sigma_{\mu} = \sigma_{\mu_{0}} + \sum_{k}^{N} \alpha_{\mu_{k}} \cos \left( k \theta_{\mu_{k}} \right) + \sum_{k}^{N} \beta_{\mu_{k}} \sin \left( k \theta_{\mu_{k}} \right) \quad \left[ C/m^2 \right],
\]

(2)

where N+1 describe number of Fourier coefficients \( \alpha_{\mu_{k}} \), and \( \beta_{\mu_{k}} \), as described in Fig. 2. With a large number of coefficients it can be assumed that the charge on the wire perimeter is divided on the 2N+1 point charges and that each point participates in

![Fig. 1 Cross-section of the transmission line wires used for CAN-bus](image1)

![Fig. 2 Illustration of per-unit-length parameters determination by moment method.](image2)
the creation of potential of all wires. Additionally, if logarithm in (1) will be transformed to power series then the algebraic equations system can be obtained instead of integral equations. They can be wrote down in matrix form:

\[
\begin{bmatrix}
\Phi_i \\
\delta_i \\
\delta_m
\end{bmatrix} = 
\begin{bmatrix}
D_{00} & \cdots & D_{0m} \\
\vdots & \ddots & \vdots \\
D_{m0} & \cdots & D_{mm}
\end{bmatrix}
\begin{bmatrix}
A_i \\
A_j \\
A_m
\end{bmatrix},
\]

(3)

\(\Phi_i\) is the potential vector of \(i\) – wire while \(A_i\) is weight coefficients vector which allows to determine of charge distribution on \(i\) - wire. Assuming \(\Phi_0 + \Phi_n\) potential values for system of neutral character and making inversion of \(D\), the values of \(\alpha_{ik}\) coefficients are reached. They show the role of particular point charges in total wire potential:

\[
A = D^{-1}\Phi, \text{ or }
\]

\[
\begin{bmatrix}
A_i \\
A_j \\
A_m
\end{bmatrix} = 
\begin{bmatrix}
B_{00} & \cdots & B_{0m} \\
\vdots & \ddots & \vdots \\
B_{m0} & \cdots & B_{mm}
\end{bmatrix}
\begin{bmatrix}
\Phi_i \\
\Phi_j \\
\Phi_m
\end{bmatrix},
\]

(4)

where \(B = D^{-1}\).

From resulting matrix it is possible to get so-called generalised capacitance matrix:

\[
C'_{ij} = 2\pi \varepsilon_i \sum_{m=1} B_{ij},
\]

(5)

Generalised capacitance matrix is the basis for another calculations which allow to calculate of per-unit-length capacitance matrix for consider conductors system (with selected one or more reference wire). For example, to determine the capacitances for wires placed over ground plane the calculations are carried out for \(2m+1\) wires where half of them are so-called mirror wires. After the adequate transformation (using \(\Phi_1 = -\Phi'\) relation) the result matrix of per-unit-length capacitances is obtained. The general capacitance matrix is at least one order less than result matrix.

When each conductor of wire is surrounded by insulation coat with \(\varepsilon_r\) permittivity, the additional effect related to dielectric polarisation should be taken into account in calculations. As the result of electrostatic induction the summarised charge accumulated on the \(\mu\)-wire represents difference between free charge on wire surface and induced charge on coat surface. It leads to some modification of matrix equations which describe the dependence between \(\phi\) and \(\alpha\):

\[
\begin{bmatrix}
\Phi_i \\
\delta_i \\
\delta_m
\end{bmatrix} = 
\begin{bmatrix}
D_{11} & D_{12} \\
D_{21} & D_{22}
\end{bmatrix}
\begin{bmatrix}
A_i \\
A_j
\end{bmatrix},
\]

(6)

where \(A^*\) is potential coefficients matrix for charges induced in dielectric. Taking into consideration the boundary conditions and their influence on value of \(\phi\) potential the \(D\) matrix is extended with additional sub-matrixes concerned with polarisation of dielectric coat. Making \(D'\) matrix inversion and multiplying it by potential vector the Fourier coefficients matrix for wire and coat is obtained. Using elements of \(D'^{-1}\) matrix the general matrix of capacitances is determined (similarly to the system with homogenous dielectric) and - after transformation - its result matrix. With assumption that only transverse electromagnetic wave is propagated in the transmission lines of consider bus it is easy to determine the inductance and leakage matrix on the basis of obtained capacitance matrix:

\[
L = H_0 \varepsilon_0 C_0,
\]

(7)

where \(C_0\) is \(C\) matrix determined at an assumption \(\varepsilon_r = 1\), and:

\[
G = 2\pi \cdot f \cdot tg\delta(f) \cdot C,
\]

(8)

when \(tg\delta(f)\) is determined for \(f\) frequency or for its harmonics.

### Capacitance MATRIX, pF/m

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On the basis of above-mentioned considerations the program (in MathCAD environment) was elaborated. It allows determining per-unit-length line parameters based on their configuration and measurements of fundamental geometrical dimensions (Fig.3). The measurements were made for several sections to eliminate of wire manufacturing inaccuracy. The obtained results were verified by measurements using RLC-bridge and necessary calculations. They confirmed the correctness and calculation accuracy of the elaborated program.

3. Multiconductor transmission line equations

With the aim of estimation of the effects of complicated phenomena appearing in transmission lines and for determination of the voltages and currents at the front and end of line the differential equation of transmission line should be solved in the general case (Fig. 4):

\[
\begin{align*}
\frac{d}{dz} \mathbf{u}(z,t) &= \mathbf{R} \cdot \mathbf{i}(z,t) + \mathbf{L} \frac{d}{dt} \mathbf{i}(z,t) \\
\frac{d}{dz} \mathbf{i}(z,t) &= \mathbf{G} \cdot \mathbf{u}(z,t) + \mathbf{C} \frac{d}{dt} \mathbf{u}(z,t)
\end{align*}
\]

Their analytical solution is possible only for the simplest configurations considered as special cases (lossless line, short-circuit state, open line, load with characteristic impedance). When the reliable results are required (real line parameters cannot be omitted) the numerical methods are used in most cases (for example Branis method used in PSPICE program). Additional difficulty in CAN-bus lines is disadvantageous coupling between wires due to their proximity, manufacturing asymmetry and careless placement. They cause possibility of the mutual generation and penetrating of disturbances. The signal propagation analysis in differential lines of CAN-bus (transmitted non-sinusoidal signals with high level of harmonics, often operated with impedance mismatch and made of many segments and stubs in the form of cables loaded with different impedances) practically is possible only with the use of informatics tools. In the considered case - based on previous investigations in area of transmission lines modelling - the operator method has been selected (Laplace transform of telegraph equation). Its idea lays on conversion from time to frequency domain, solving of differential equations for assumed harmonics and subsequently making the conversion to the time domain.

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\[
\begin{align*}
\mathbf{u}(z,s) &= \mathbf{L}(\mathbf{i}(z,s)) \\
\mathbf{i}(z,s) &= \mathbf{L}^{-1}(\mathbf{u}(z,s))
\end{align*}
\]

where \( \mathbf{L} \left( \frac{\partial}{\partial t} \right) = j\omega = s \).

After substitution \( \mathbf{Z} = \mathbf{R} + j\omega \mathbf{L} \), \( \mathbf{Y} = \mathbf{G} + \frac{j\omega}{s} \mathbf{C} \) and assuming \( \mathbf{u}(z,0) = 0, \mathbf{i}(z,0) = 0 \) we can get the system of equations uncoupled in relation to voltage and current vectors:

\[
\begin{align*}
\frac{d^2}{dz^2} \mathbf{u}(z,s) &= \mathbf{Z} \cdot \mathbf{Y} \cdot \mathbf{u}(z,s) \\
\frac{d^2}{dz^2} \mathbf{i}(z,s) &= \mathbf{Y} \cdot \mathbf{Z} \cdot \mathbf{i}(z,s)
\end{align*}
\]

Such equation, as second-order differential equation with constant coefficients has a solution for one-element impedance and admittance matrices. For
multiconductor lines the solution is possible (providing that the transformation allowed to separate of equations connected by matrixes $\mathbf{Z} = \mathbf{R} + s\mathbf{L}$, and $\mathbf{Y} = \mathbf{G} + s\mathbf{C}$ will be introduced). Using transformation by analogy these matrixes can be reduced to the diagonal form:

$$
\mathbf{z} = \mathbf{T}_u^{-1} \mathbf{Z}_u \mathbf{T}_i,
$$
\text{(12)}

$$
\mathbf{y} = \mathbf{T}_i^{-1} \mathbf{Y}_u \mathbf{T}_i,
$$
\text{(13)}

Taking into account the long-lines property (product of admittance and impedance diagonal matrixes gives matrix of squares of constant propagation $\gamma^2$) we obtain:

$$
\frac{d^2}{dz^2} \mathbf{U}_m(z) = \tilde{\mathbf{a}}^2 \mathbf{U}_m(z),
$$
\text{(14)}

Knowing the general solution of such differential equations type the expression (13) can be described as algebraic equations system in a form showing here for currents:

$$
\mathbf{I}_m(z) = \exp(-\gamma_1 \cdot z) \mathbf{I}_{m1} - \exp(\gamma_1 \cdot z) \mathbf{I}_{m0}.
$$
\text{(15)}

After determination of the integral constants $\mathbf{I}_{m1}, \mathbf{I}_{m0}$ from the boundary conditions, the real currents flowing in wire system can be calculated (after reverse transformation). In similar procedure, the solution of voltage equations can be obtained. Finally, the equations have a form:

$$
\mathbf{U}(z) = \mathbf{T}_i \cdot \mathbf{I}_m(z) = \mathbf{T}_i \left( \exp(-\gamma_1 \cdot z) \mathbf{I}_{m1} - \exp(\gamma_1 \cdot z) \mathbf{I}_{m0} \right)
$$
\text{(16)}

where $\mathbf{Z}_c$ is characteristic impedances matrix of transmission line system.

The presented – in broad outline – method of telegraphs equation solution is the basis of elaboration of the algorithm and analysis program of signal propagation process in transmission lines.

Again, the numerical algorithm was implemented in Math-CAD program. The determination of conversion matrix $\mathbf{T}_u$ and matrix of propagation constants $\gamma$ is relatively easy thanks to functions built in the program. On their basis and with knowledge of the boundary conditions for voltages (currents) and impedance (admittance) for sources and loads in particular lines the integral constants can be determined. Finally, the searched values of voltage and current can be calculated from relations (15) and (16).

Finding of system response in the frequency domain $U(\omega)$ on defined input function requires multiplying of the system transmittance $H(\omega)$ by frequency representation of the input signal $U_0(\omega)$. Accordingly to the assumed discrete frequency spectrum of the analysed signal the module and phase $|H(\omega)|$ and $\phi(\omega)$ of transfer function can be calculated (for frequencies of harmonics and their number $N_h$). Those values allow to calculate the signal representation in time domain using inverse Laplace transform:

$$
|U(t)| = \mathbf{L}^{-1} \cdot U(\omega) = H(0) \cdot U_0(0) + \sum_{n=1}^{N_h} |H(n \frac{2\pi}{T})| \cdot |U_0(n \frac{2\pi}{T})| \cdot \cos\left(n \frac{2\pi}{T} t + \phi(n \frac{2\pi}{T}) + \phi(\omega)\right).
$$
\text{(17)}

4. Transmission line networks

The above-mentioned considerations are insufficient in the case of program creation for determination of signal propagations in CAN-bus. Such device is the heterogeneous system, consists of many segments having different properties related to signal propagation. It results from the characteristic of CAN system which allows different alternative line creation, topology adopted for users, random bus node placement, etc. From the point of view of the propagation parameters simulation in such transmission line, the most important are following aspects:

- location of nodes in particular points of
network in relation to current requirements; it causes that system is the connection of many varied transmission lines with nodes which play the role of loads in particular network segments;

• use of different network topologies (bus, star, mixed systems);

• node connections using branches of the specific length, connection with measuring and service devices, dynamic adding and removing of nodes;

• possible application of different types of wires differently placed in the same network.

Taking into account those factors it should be noted that CAN-bus can be divided into N-segments. Each segment can be represented by homogenous model of transmission line with parameters possible to be established. The segments examples are: line section (placed in air) from terminator to the end of line made of tape, segment of the same tape placed directly on grounded metal construction till to the node, segment going from one node to the next node, etc. The particular segments connection will have its character dependent on the type of line parts. It can be e.g.: direct segment connection with different per-unit-length parameters, impedance or admittance of the k-node or terminator. It is worth to note that in the similar way the signal propagation can be analysed in so-called twisted cable. Such wire is considered as series connection of homogenous transmission lines (with subsequent loops).

The voltages and currents for adequate segments of presented lines can be made dependent on each other. For each node the equation is as follows:

\[ \ldots + Y_i^n \cdot I_i^n + Z_i^n \cdot V_i^n + Y_j^n \cdot I_j^n + Z_j^n \cdot V_j^n + \ldots = P^* \]  

(17)

Making a number of transformations those expressions can be presented in matrix form:

\[
\begin{bmatrix}
Y_i^n + Z_i^n \cdot Y_{i,0} + (Y_j^n + Z_j^n \cdot Y_{j,0}) + (Z_j^n \cdot Y_{j,0}) + \ldots \\
Y_j^n + Z_j^n \cdot Y_{j,0} + (Y_j^n + Z_j^n \cdot Y_{j,0}) + (Z_j^n \cdot Y_{j,0}) + \ldots
\end{bmatrix}
\begin{bmatrix}
Y_i^n \\
Y_j^n
\end{bmatrix}
= \begin{bmatrix}
P^* - Z_i^n \cdot I_{i,0} - Z_j^n \cdot I_{j,0}
\end{bmatrix}
\]

(18)

From those relations the boundary conditions for particular line segments can be found by analogical transformation as described above.

5. CAN-bus transmission line calculations

With the aim of verification of the performed investigations the voltage measurements were made in test circuit of CAN-bus together with calculations of adequate waveforms with the help of elaborated programs. The experiments were carried out for line made from ribbon wires (length 24m - Fig.3) with trapezoidal pulse (Fig.6).
**Fig. 6** Shape of test trapezoidal pulse: \( T=1\mu s, U_m=1V, t_r=t_f=8V/\mu s \) (Ch1) and \( t_r=t_f=6V/\mu s \) (Ref1) (generated by HP-33120A generator with load 50Ω).

It was assumed that the test line was loaded in 2 points: in half and at the end of bus-length (load as for 10 parallel connected typical nodes - \( R=2k\Omega \) and \( C=200pF \)).

**Fig. 7** Voltage waveforms of unloaded mismatched line (\( t_p=60ns \))

**Fig. 8** Voltage waveforms of unloaded matched line (\( t_p=120ns \))

**Fig. 9** Voltage waveforms of matched line loaded at the end by 10 nodes (\( t_p=140ns \))
Fig. 10 Voltage waveforms of matched line loaded at the end by 10 nodes and in half of length by 10 nodes (tp=170ns)

The programmable generator HP-33120A was used as the signal source (Fig. 6). The trapezoidal pulse was repeated with frequency f=500 kHz which corresponds to one bit of High-Speed CAN system transmitted with the speed 1MB/s.

The waveforms of signals in CAN-bus at the front (1) and end (2) of CAN_H line have been measured (upper part) and calculated (lower part of figures 7-10). Simulations and measurements have been provided for following conditions: unloaded and mismatched line (Fig. 7), unloaded and matched line (Fig. 8), matched line loaded at the end by 10 nodes (Fig. 9), matched line loaded at the end by 10 nodes and in half of length by 10 nodes (Fig. 10).

The obtained results – in spite of a certain imperfection (especially regarding the program for signal propagation analysis) – allow drawing some conclusions about the correctness of assumptions in realizing the configuration of CAN controllers. First of all, the signal propagation times can be determined between particular nodes which – as show the received results – are dependent on number of nodes and their placement. It plays a very important role for proper system configuration. It is also possible to calculate the voltage drops in transmission lines and ground wire to determine if the received values are not too large to provide adequate voltage levels for proper transceivers operation.

By analysis of the characteristic impedance matrices of transmission lines the selection of differential mode termination can be modified and – if necessary – proper common mode termination can be introduced.

6. Conclusions

The results of investigations have proofed the possibilities of parameters calculation, required to choose relevant CAN controller configuration during the design. Elaborated programs allow calculation of time of the signal propagation and voltage drops in the lines in relation to wire types and their configuration. It allows checking the correctness of applied assumptions and indicates the modification areas. Those calculations will be the most useful in critical cases (max. speed, length, number of nodes). Actually the programs are suitable rather for the easiest practical cases, as it was presented in the paper. However conducted investigations are aimed to extend the CAN-bus analysis on any real configuration.

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